AN INVESTIGATION OF THE RELATIONSHIP BETWEEN THE NATURAL PITCHING PERIOD OF A SHIP AND THE HULL FORM PARAMETERS

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by

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ABSTRACT

AN INVESTIGATION OF THE RELATIONSHIP BETWEEN THE NATURAL PITCHING PERIOD OF A SHIP AND THE HULL FORM PARAMETERS

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Submitted to the Department of Naval Architecture and Marine Engineering on May 26, 1958 in partial fulfillment of the requirements for the degree of Naval Engineer and the Degree of Master of Science in Naval Architecture and Marine Engineering.

The purpose of this thesis was to investigate the relationship between the natural pitching period of a ship and the hull form parameters. A theoretical relationship was developed by using simple geometric forms as approximations for the sectional area curve and the waterplane shape. This resulted in the formula:

$$Tp/\sqrt{L} = Ck\sqrt{\frac{\Delta}{(.01L)^3}}$$

where Ck is a function of Cp, Cw, L/H, L/B and the ratio of the length and beam of the submerged volume to the waterline length and beam. The accuracy of the theoretical relationship was determined by predicting the period-length ratio and comparing it with the values computed for 79 ships of various types and 6 hull forms. Good agreement was obtained for the great majority of cases.

The theoretical relationship was used to show that the important methods of decreasing the period-length ratio at any displacement-length ratio are to increase the draft and the waterplane area. Due to past practices in ship design an empirical relationship for the period-length ratio versus displacement-length ratio exists. It is also possible to markedly increase the natural pitching period to a value where super-critical operation becomes feasible at normal speed-length ratios. This can be done by decreasing the waterline length and beam in relation to that of the maximum dimensions of the submerged volume.

Further study should investigate the effect of ship velocity on the coefficient of accession to inertia. The effect of damping, coupling to heave and large angles of pitch on the natural pitching period should also be investigated. A feasibility study of a vessel for supercritical operation is also recommended.

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INTRODUCTION

The pitching motion of ships has been receiving increasing attention as the result of studies in the field of seakeeping. One of the more recent theories concerning the seakeeping of ships is that presented by E.V. Lewis [7]. This theory is based on recent oceanographic studies and several assumptions which appear to be valid. The results of the oceanographic studies indicate that a rough sea can be closely approximated by the superposition of seas of all wave lengths.

In the development of his theory, Lewis assumes that a ship will be in a critical condition as far as pitching motions are concerned when heading into a sea which has waves of length equal to that of the ship and with the period of encounter with this wave equal to the ship's natural pitching period. This synchronism would result in magnification of the pitching motion of the ship. In this condition, the ship will usually be forced to decrease speed or change course to prevent slamming or the shipping of seas over the bow. Since the length of the wave determines the speed of the wave, the natural pitching period of the ship determines the speed at which the ship can move before it reaches this critical condition of operation.

In further developing this theory, Lewis [7] uses a rough approximation to show that the $T_p/\sqrt{L}$ ratio for a ship varies with the displacement-length ratio. This approximation has been questioned and the data upon which it is based appears to be inconclusive. The purpose of this thesis is to further investigate the relationship between the natural pitching period of a ship and its hull form parameters and thereby clarify this questionable point in the theory presented by Lewis [7].

The formula for computing the natural pitching period of a ship is derived from the simple equation of pitching motion:

$$I_y \frac{d^2\theta}{dt^2} + \rho g J_y(\theta) = 0 \qquad (1)$$

This equation implies several basic assumptions which are not necessarily valid. They are:

1. The damping of motion of the hull has a negligible effect on the natural pitching period.
2. The coupling of pitching motion to heave has a negligible effect on the natural pitching period.

3. The velocity of the ship has a negligible effect on the natural pitching period.
4. The motion in pitch is limited to small angles.

The damping of motion has been shown to have only small effect on the natural undamped period in studies of simple harmonic motion. This holds true as long as the magnitude of the damping is small which is the case in the pitching motion of a ship. This assumption is accepted for this study since no contrary information is available.

The coupling of motion between pitch and heave is a result of the assymmetry of the hull form and the forward velocity of the ship. A great deal of study remains to be done in the field, but to date, there is no indication that the effect of coupling causes an appreciable effect on the natural pitching period as computed by the simple equation (1). This effect, therefore, has been neglected in this study.

There is definitely some effect on the natural pitching period due to the forward velocity of the ship. This velocity can be expected to affect the correction for the accession to inertia of the entrained water, the damping of the hull form and the coupling of motion between heave and pitch. To date, the magnitude of the change in the natural pitching period due to the ship's velocity is unknown. Any correction for this effect can only be neglected until much more work is done in this field.

The need for a correction for the accession to inertia due to the entrained water was first stated by Kriloff [5] . Methods of determining this factor have been given by F. M. Lewis [8] , by G. Vedeler [14], and by Weinblum and St. Denis [16] . This last correction is computed for a completely submerged ellipsoid, but apparently gives satisfactory results for surface ships. It is favored by J. C. Niedermair [10] . This form of the correction for accession to inertia has been used in this study.

A further ramification of the theory presented by Lewis [7] is the fact that ships do not have to operate in the sub-critical range previously described. If the speed of a ship is increased above the critical speed, the ship will enter into synchronism with waves of length greater than the ship's length. This condition can be even worse than the critical condition if the waves of this length have greater energy and thereby represent a greater exciting force. If the speed of the ship can be increased further, however, the ship will eventually enter into synchronism

with a wave length that has very little energy and therefore, very little exciting force. Lewis [7] called this "supercritical" operation. To achieve this condition, a large $T_p/\sqrt{L}$ ratio is advantageous. In the range of normal $T_p/\sqrt{L}$ ratios, a very high speed-length ratio is required. This study investigates abnormal hull forms which are designed to have a very large $T_p/\sqrt{L}$ ratio for the purpose of attaining "supercritical" operation at lower speed-length ratios.

<u>PROCEDURE</u>

The formula for the natural pitching period of a ship can be derived from the simple, undamped, uncoupled equation for pitching motion:

$$I \frac{d^2\theta}{dt^2} + \rho g J (\theta) = 0 \tag{1}$$

It is, in the usual form:

$$T_P = 2\pi \sqrt{\frac{I}{\rho g J_y}} \tag{2}$$

where I is the total mass moment of inertia of the ship and J_y is the moment of inertia of the waterplane area. I is usually expressed as:

$$I = I_y + k_{yy} I_v \tag{3}$$

where:

I_y is the mass moment of inertia of the weight

I_v is the mass moment of inertia of the displaced volume

k_{yy} is the coefficient of accession to inertia.

therefore, $T_P = 2\pi \sqrt{\frac{I_y + k_{yy} I_v}{\rho g J_y}}$ (4)

In normal terms:

$$I_y = \rho \nabla K_y^2$$

$$I_v = \rho \nabla K_v^2$$

where K_y and K_v are the gyradii of the load and volume respectively. In addition,

$$k_y = {}^{K_y}/_{L} \quad \text{and} \quad k_v = {}^{K_v}/_{L}$$

and $\nabla = C_B$ BHL

where B,H and L are the dimensions of the displaced volume used in computing C_B.

$$J_y = \alpha_L B_w L_w^3$$

where B_w = beam of waterplane

L_w = length of waterplane

Substituting the previously defined relationships in the formula for T_P:

$$T_P = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{\rho C_B BHL^3 k_y{}^2 + k_{yy} \rho C_B BHL^3 k_v{}^2}{\rho B_w L_w{}^3 \alpha_L}}$$

$$\text{and } T_P = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{BL^3}{B_w L^3{}_w}} \sqrt{k_y{}^2 + k_{yy} k_v{}^2} \sqrt{\frac{C_B}{\alpha_L}} \sqrt{H}$$

There can be no exact relationship between k_y and k_v since k_v is fixed by the shape of the displaced volume while k_y can be varied by changing the location of loads within the ship. In practice, however, major changes of shape or loading introduce only minor changes in k_y and k_v. In addition, similar types of ships will tend to be loaded in a similar manner and the location of the load will be limited by the shape of the hull. It will, therefore, be assumed that an approximate empirical relationship exists such that

$$k_y{}^2 = c\, k_v{}^2$$

Appendix A gives values of c for various ships.

$$T_P = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{BL^3}{B_w L_w{}^3}} \sqrt{C + k_{yy}} \sqrt{C_B H} \sqrt{\frac{k_v{}^2}{\alpha L}}$$

and

$$\frac{T_P}{\sqrt{L}} = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{BL^3}{B_w L_w{}^3}} \sqrt{(C + k_{yy}) \quad C_B \frac{H}{L}} \sqrt{\frac{k_v{}^2}{\alpha L}} \qquad (5)$$

In Appendix A an approximate relationship for $\sqrt{\frac{k_v{}^2}{\alpha L}}$ is developed for all ships as

$$\sqrt{\frac{k_v{}^2}{\alpha L}} = f = \sqrt{\frac{(2C_P{}^2 - 2C_P + 1)}{C_W(2C_w{}^2 - 2C_W + 1)}} \qquad (6)$$

$$\frac{T_P}{\sqrt{L}} = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{BL^3}{B_W L_W{}^3}} \sqrt{(C + k_{yy})} \sqrt{\frac{C_B H}{L}} \quad (f)$$

Defining

$$n = \sqrt{\frac{BL^3}{B_W L_W^{\;3}}} \qquad (7)$$

$$\frac{T_P}{\sqrt{L}} = \frac{2\pi}{\sqrt{g}} \sqrt{\frac{C_B H}{L} \; (C+k_{yy})} \;\; (f)\; (n)$$

Formula (8) represents a straight line approximation to Fig. 6 for $L/_B \geqq 5$.

$$k_{yy} = .440 \; {}^{B}/_{H} \; + .0233 \;\; {}^{L}/_{B} - .285 \qquad (8)$$

Assuming C = 1.00

$$\sqrt{\frac{C_B H}{L} \; (C+k_{yy})} = \left[C_B(\frac{H}{L}) \; (.440 \; \frac{B}{H} + .0233 \; \frac{L}{B} + .715) \right]^{1/_2}$$

and $\frac{C_B H}{L} = \frac{\nabla}{L^3} \; (\frac{L}{B})$

$$\sqrt{\frac{C_B H}{L} \; (C+k_{yy})} = \left[\frac{\nabla}{L^3} \;\; (.715 \; \frac{L}{B} + .0233(\frac{L}{B})^2 + .440 \; \frac{L}{H} \;) \right]^{1/_2}$$

and $\sqrt{\frac{C_B H}{L} \; (C+k_{yy})} = \sqrt{35x10^{-6}} \;\; \sqrt{.715 \; \frac{L}{B} + .0233 \; (\frac{L}{B} \;)^2 + .440 \; \frac{L}{H}} \; \sqrt{\frac{\triangle}{(.01L)^3}}$

and $\sqrt{\frac{C_B H}{L} \;\; (C+k_{yy})} = \sqrt{35x10^{-6}} \;\; \sqrt{\frac{\triangle}{(.01L)^3}} \;\; (h)$

where h is defined as

$$h = \left[.715 \; \frac{L}{B} + .0233 \; (\frac{L}{B} \;)^2 + .440 \; \frac{L}{H} \right]^{1/_2} \qquad (9)$$

$$\frac{T_P}{\sqrt{L}} = \frac{2\pi}{\sqrt{g}} \sqrt{35x10^{-6}} \; (f) \; (h) \; (n) \; \sqrt{\frac{\triangle}{(.01L)^3}}$$

$$\frac{2\pi}{\sqrt{g}} \sqrt{35 \times 10^{-6}} = 0.00655$$

and:

$$\frac{T_P}{\sqrt{L}} = C_K \sqrt{\frac{\Delta}{(.01L)^3}} \tag{10}$$

where C_K is defined as:

$$C_K = 0.00655\ (f)\ (h)\ (n) \tag{11}$$

Using formula (10) values of C_K were determined for a large number of ships from $T_P/\sqrt{L}$ and the displacement length ratio. Values of C_K were also predicted by formula (11).

FIGURE I
PREDICTION OF C_K ERROR DISTRIBUTION

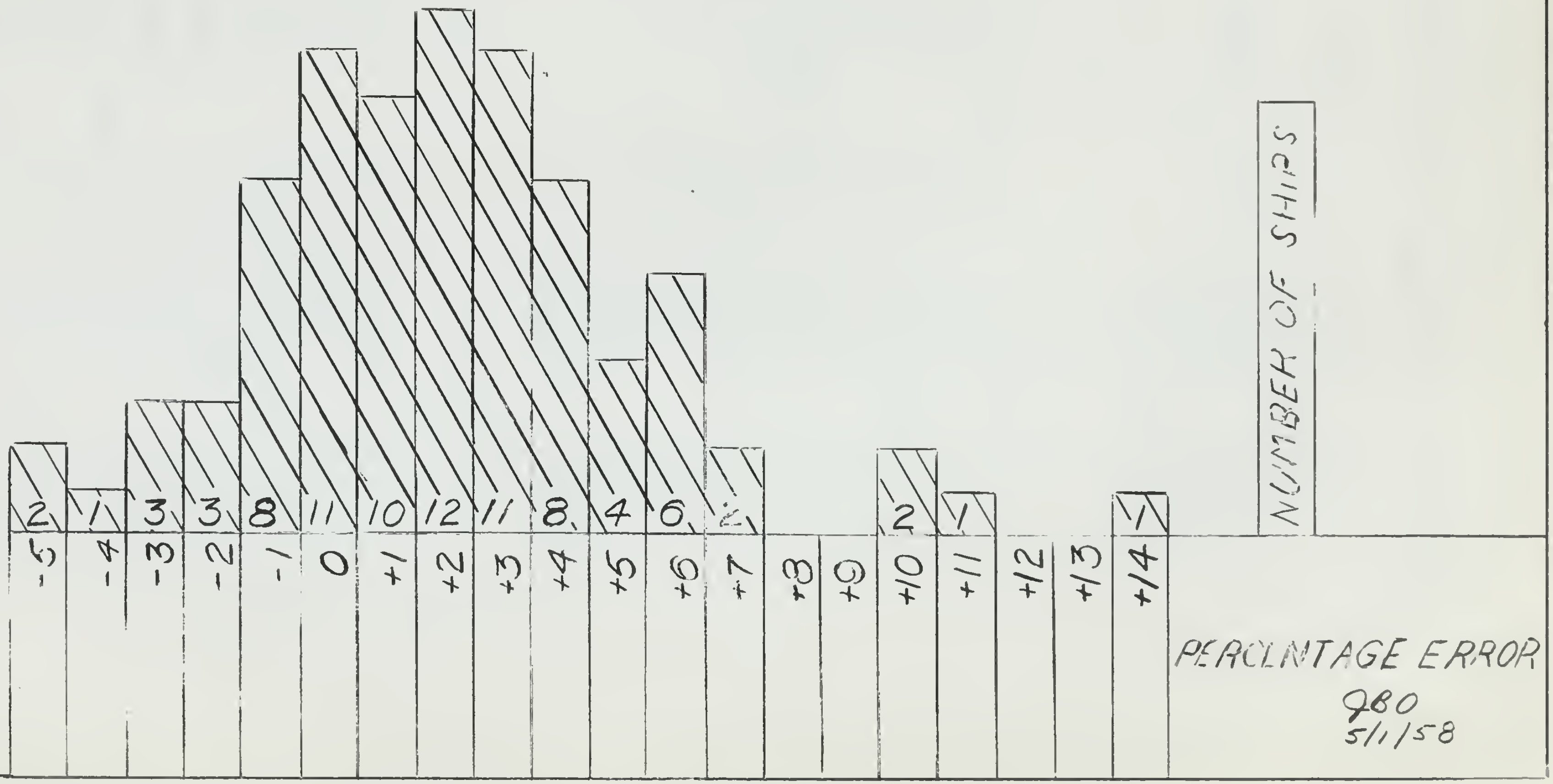


Table I

Specific Ships by Percentage Error in Predicting C_K

-5%		-4%		-3%	
CVL-48	No. 17	DD-931	No.13	DD-348	No. 3
BB-43	No. 50			LST-1153	No. 26
				Passenger	No. 33

-2%		-1%		0%	
DD-692 (LH)	No. 9	CV-Yorktown	No.6	DD-445	No. 5
CA-32	No. 14	CL-40	No. 8	CA-68	No. 7
LST-1	No. 36	CV-32	No. 16	DD-692 (SH)	No. 12
		CV-2	No. 19	CL-144	No. 15
		Passenger	No. 22	Tanker	No. 56
		Passenger	No. 29	Tanker	No. 57
		LSM-1	No. 37	Tanker	No. 60
		AE-21	No. 45	Cargo	No. 64
				Cargo	No. 68
				Tanker	No. 69
				Ellipsoid	E-1

+1%		+2%		+3%	
DD-356	No. 2	CL-55	No. 10	CA-139	No. 11
PG-50	No. 23	CB-1	No. 18	Barge	No. 24
Passenger	No. 39	BB-61	No. 25	Cargo, Ore	No. 27
Cargo C-4	No. 43	CM-5	No. 28	Passenger	No. 41
Passenger	No. 44	Passenger	No. 30	BB-Penn	No. 48
Tanker	No. 62	Cargo	No. 52	Cargo	No. 49
Cargo	No. 66	Motor Sailer	No.54	BB-Del.	No. 53
Ellipsoid	E-2	Tanker	No. 55	Cargo	No. 58
Ellipsoid	E-3	Cargo	No. 63	Cargo - C-2	No.59
Ellipsoid	E-4	Cargo	No. 65	Tug	No. 67
		Tug	No. 74	Tug	No. 78
		Ellipsoid	E-6		

+4%

Cargo	No. 21
Passenger	No. 31
Passenger	No. 32
Cargo	No. 34
Cargo	No. 46
Dredge	No. 71
Trawler	No. 73
Ellipsoid	E-5

+5%

CV-Ranger	No. 1
Ferry	No. 35
Tanker	No. 61
Trawler	No. 70

+6%

CVB-42	No.20
AD-32	No. 40
Tanker	No.47
Ferry	No. 51
Tug	No.75
Tug	No.79

+7%

CL-51	No. 4
BB-55	No. 38

+10%

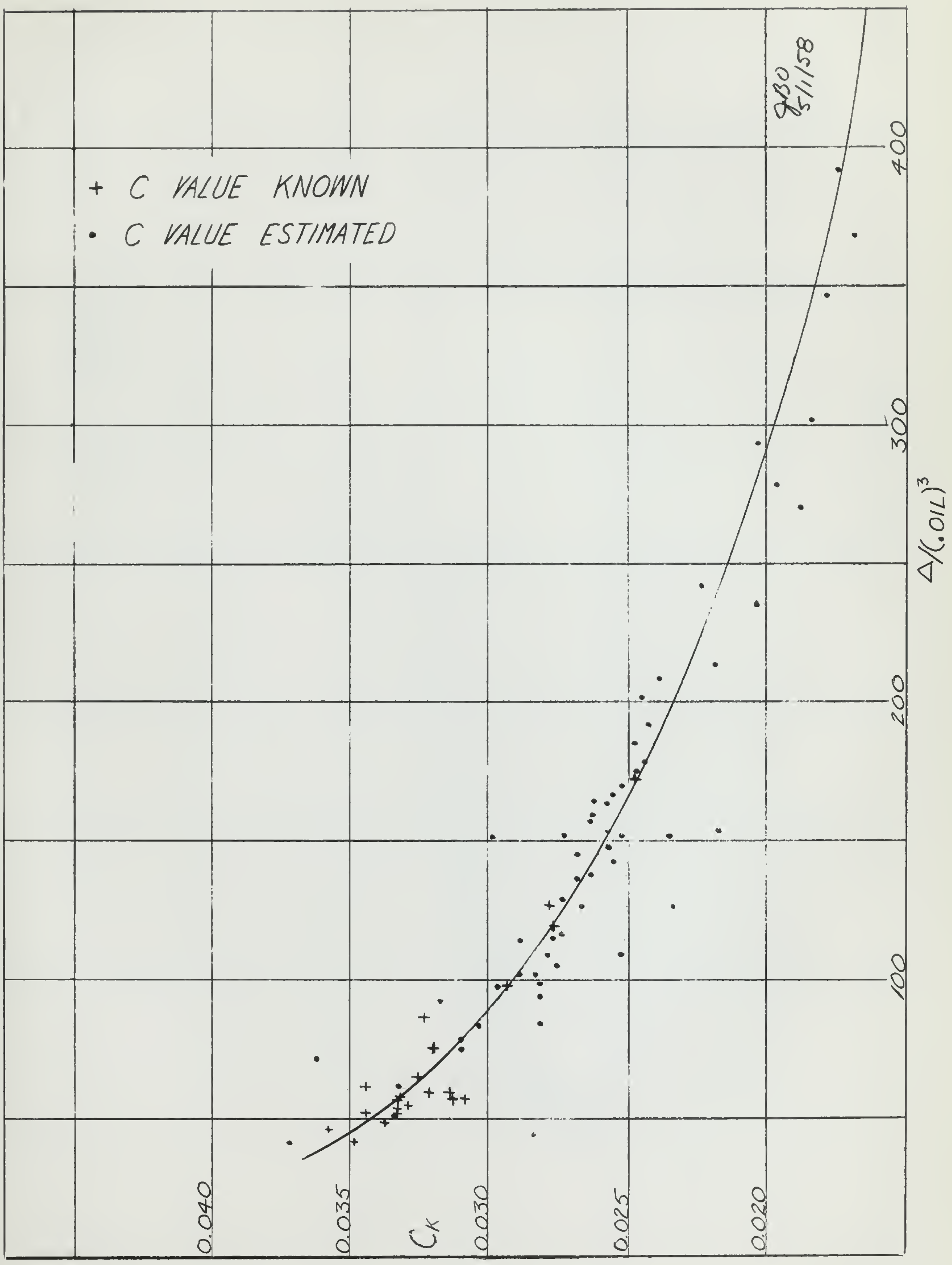
Trawler	No. 72
Tug	No. 76

>+10%

Launch	No.42
Tug	No. 77

FIGURE II

C_K vs. $\Delta/(.01L)^3$



FIGURF III

$T_p/\sqrt{L}$ vs. $\Delta/(.01L)^3$

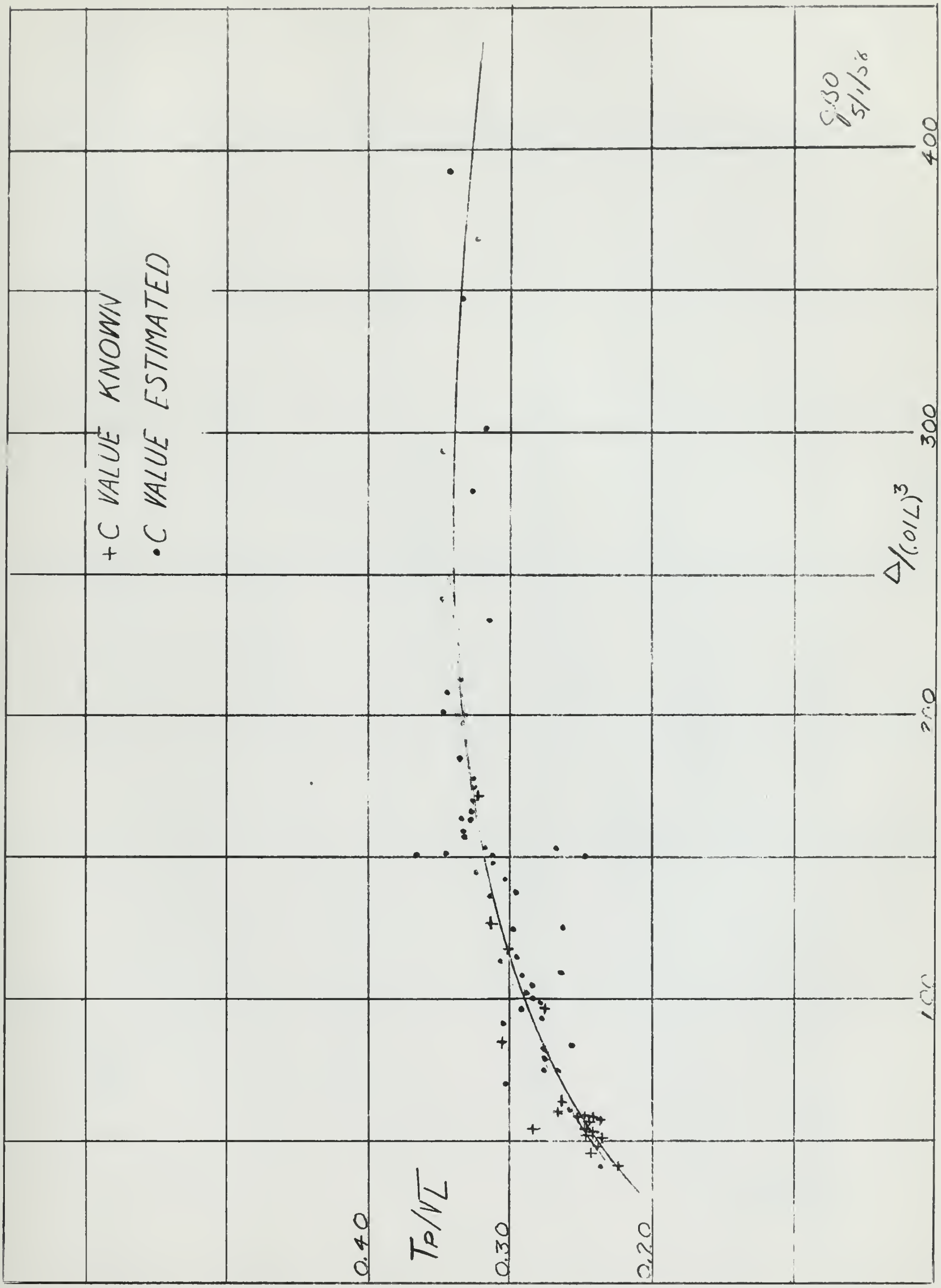


FIGURE IV

CONDITIONS OF SYNCHRONOUS PITCHING

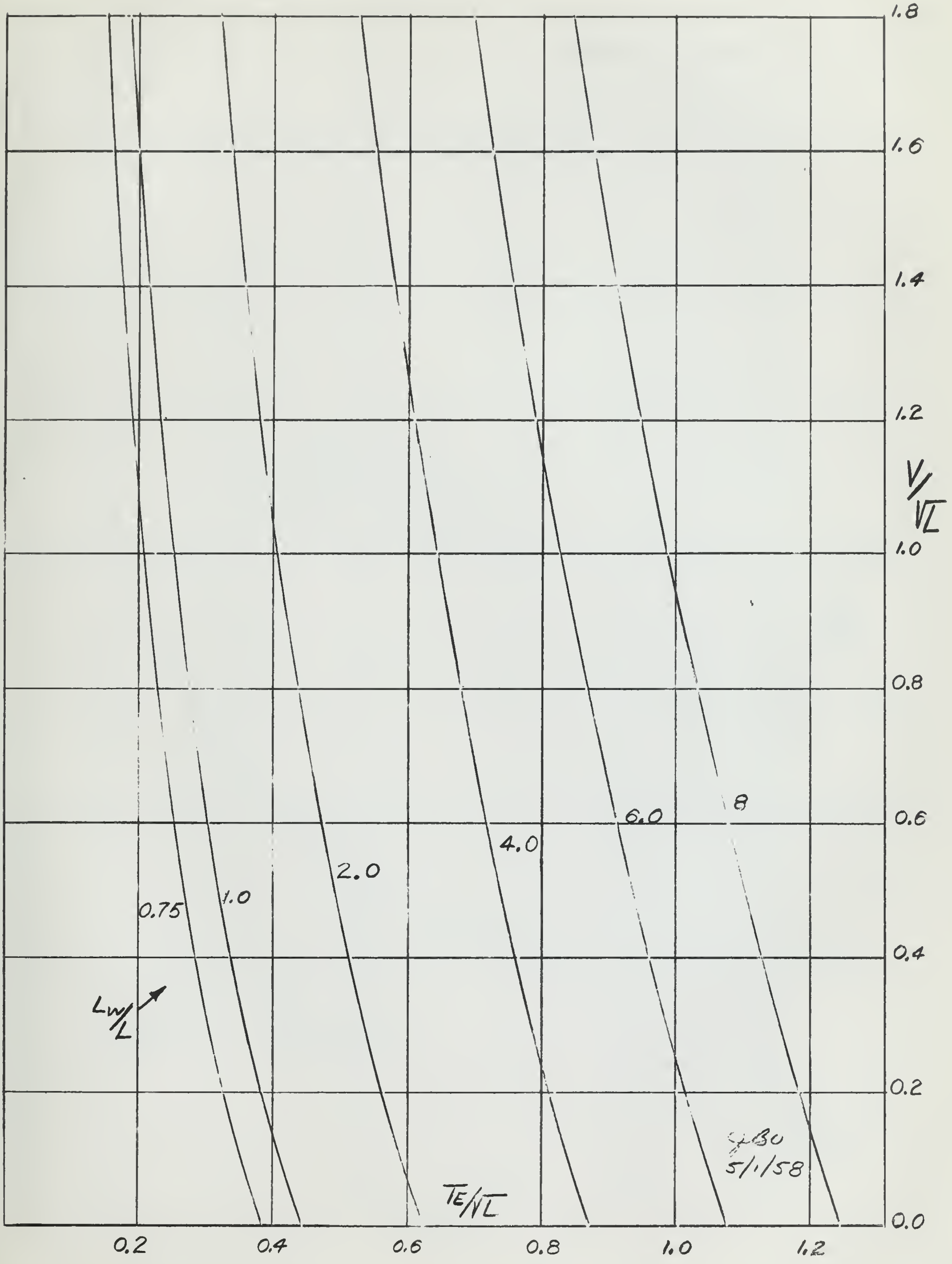
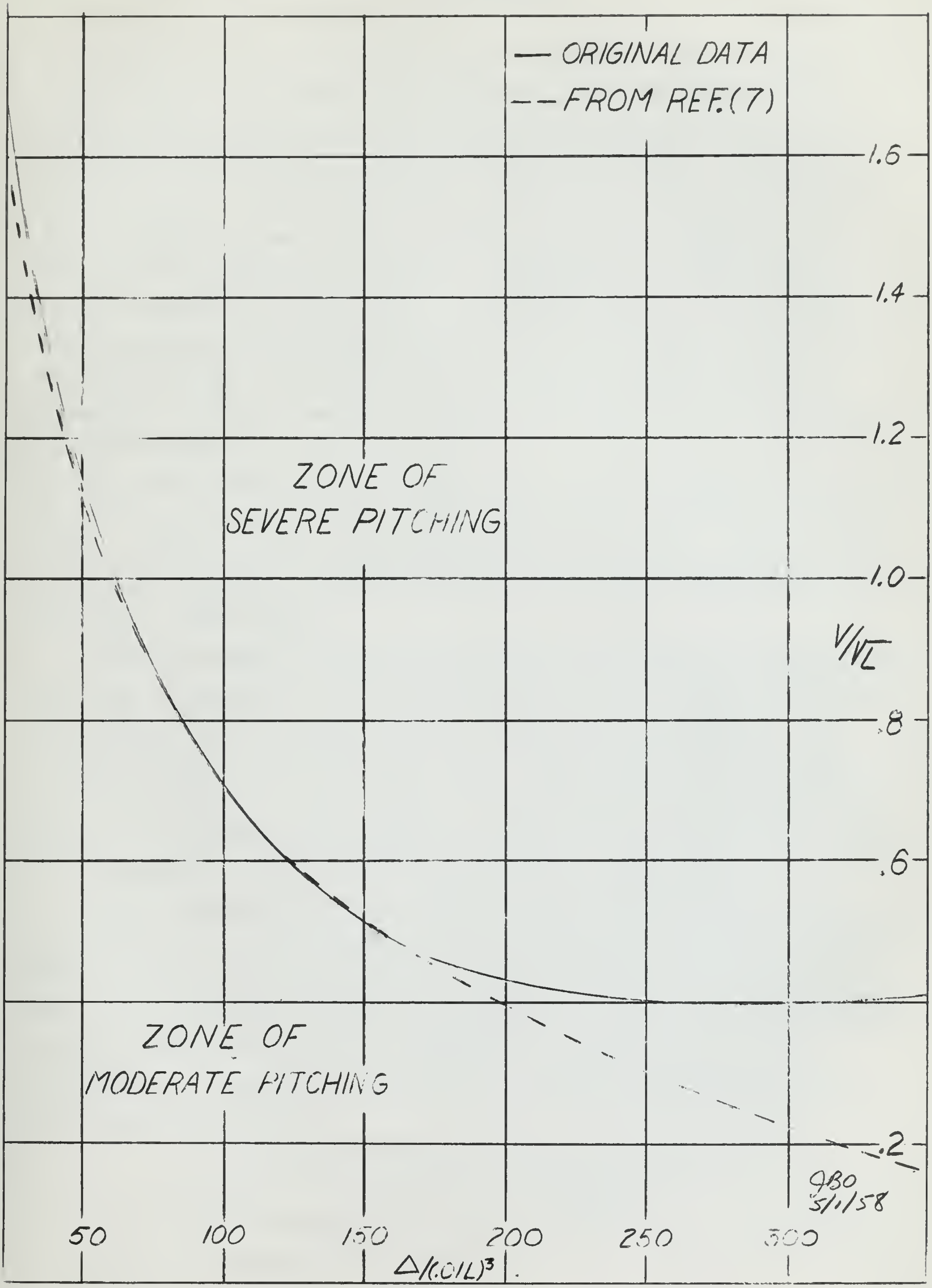


FIGURE V

SYNCHRONOUS PITCHING IN REGULAR HEAD SEAS



DISCUSSION

The formula (11) for predicting the constant of proportionality, Ck, was tested on seventy-nine actual vessels and six simulated hull forms based on ellipsoids. The predicted values of Ck for these ships were obtained with the aid of figures VII, VIII, and IX. In all of the actual ships the value of the factor n was equal to one or very closely so. The value can not be less than one. In the six ellipsoids, the value of n was computed and applied. The actual or computed value of Ck for these ships was obtained by computing the value of the natural pitching period by the standard formula (4). The computed value of Ck was then obtained and compared to the predicted value and the error calculated. Figure I and Table I show the error distribution for these eighty-five cases.

As can be seen from Table I, the error in predicting Ck and therefore $Tp/\sqrt{L}$, is within the range of -5% to +7% for the great majority of the cases. Only four of the vessels exceed this error range. Each of these four cases represents ships of very high displacement-length ratio and very short actual length.

Whenever weight data was not available for any ship, an approximation was used to estimate Iy in order to compute $Tp/\sqrt{L}$. This assumption was that similar types of ships will be loaded in a similar manner. An average value of c was obtained for each type of ship for which weight data was available. These values of c were applied to similar ships for which no weight data was available. A value of c equal to 1.00 was used in the cases where there was a complete lack of weight data.

In the nonmenclature used in this study, a positive error in Ck represents a lower $Tp/\sqrt{L}$ ratio than was predicted. As can be seen from Figure I, the error distribution is slightly skewed to a positive error. The errors represented by this table can be accounted for by the approximations used in developing the formulas for f and h (6)(9). The derivation of these factors is covered in Appendix A and includes the following approximations:

a. The sectional area curve can be represented by two symmetrical triangles and a rectangle.
b. The waterplane shape can be represented by two symmetrical triangles and a rectangle.

c. The ratio of the mass moment of inertia of the load to that of the displaced volume is equal to one.

d. The value of $(C_w \bar{l}^2 - d^2)$ is negligible.

These conditions can only be approximately true for any ship and therefore the predicted value of Ck can be only approximately accurate. The small spread of the errors in these eighty-five cases however, indicates that the formula for Ck (11) can be used to predict the result of a change in the hull form parameters on the natural pitching period.

In accordance with the theory presented by Lewis [7], present day ships operate in a subcritical condition in heavy seas. In accordance with this theory, the smaller the $Tp/\sqrt{L}$ ratio a ship has, the higher the speed-length ratio it can reach and still remain in a subcritical condition. It is, therefore, advantageous from the point of view of maintaining a higher speed in rough seas to have as low a value of $Tp/\sqrt{L}$ as possible. At any fixed value of displacement-length ratio, a lower value of $Tp/\sqrt{L}$ can be obtained by attaining a lower value of Ck. Since Ck depends on the factors f and h, the effect of the variation of the hull form parameters can be obtained from formulas (6) and (9).

These formulas, (6) and (9), indicate that the dimensions of the ship which are important in changing the value of Ck are the draft and the waterplane area. Increasing the draft and the waterplane are of a design decreases the $Tp/\sqrt{L}$ ratio. For a design with constant length, beam and displacement, an increase in the draft and the waterplane area implies an approach to V-form sections. This may be one explanation for the fact that V-form ships exhibit better seakeeping characteristics. It is more likely however, that the increased damping associated with V-form sections may be a more important consideration. This, however, does reinforce the desirability of V-form sections from a seakeeping standpoint.

In the design of most ships, the draft is limited by physical considerations. In addition, the maximum waterplane area that may be obtained is limited. It has also been design practice to attain the maximum practical draft in most sea going ships. This implies that only small variations can be expected in the value of Ck for ships at any displacement-length ratio. This is borne out by Figure II in which the

computed value of Ck for the seventy-nine ships is plotted versus the displacement-length ratio. The scatter about the empirical curve drawn through these points is definitely limited. Since a wide variety of ships was studied, this curve indicates that only minor variations in Ck can be obtained once the displacement-length ratio is fixed.

Figure III shows a plot of $Tp/\sqrt{L}$ versus the displacement-length ratio. The results obtained from the study of the seventy-nine ships are plotted on this curve and the empirical curve of Figure II is shown. This line represents the average variation of $Tp/\sqrt{L}$ with the displacement-length ratio and is strictly an empirical result of past practices in ship design. Figure III, when combined with the plot of $Te/\sqrt{L}$ versus the speed-length ratio shown in Figure IV results in a curve of the speed-length ratio versus the displacement-length ratio shown in Figure V. The curve given by Lewis [7] is also plotted on this figure for purposes of comparison. It can be seen that they agree closely except at high displacement-length ratios. The exact position of the curve at high displacement-length ratios is not conclusive due to the scarcity of data of actual ships.

In order to determine the accuracy of the factor n in formula (7), an ellipsoid of six hundred feet in length and sixty feet in diameter was studied at various drafts from thirty to fifty-six feet. As the draft increases, the value of the factor n increases due to the difference between the waterline length and beam and that of the submerged hull. The predicted values of Ck were in satisfactory agreement with the computed values. The value of Ck was of a much higher magnitude than those for normal hull forms. These results indicate that a high value of $Tp/\sqrt{L}$ can be obtained by going to abnormal hull forms.

In order to operate in a "supercritical" condition, the period of encounter with the longest significant component of a sea must be less than the ship's natural pitching period. In the data presented by Lewis [7], the wave length of the longest significant component for a forty knot, fully developed sea, is about 2400 feet. This represents a factor of four for a six hundred foot ellipsoid. From Figure IV, it can be seen that the ellipsoid with a draft of fifty-six feet can operate in this sea in the "supercritical" range with a speed-length ratio of

approximately one. This indicates that "supercritical" operation is not necessarily limited to the future but may be obtained at the present time.

The deeply submerged ellipsoid type of hull form may not be satisfactory for many reasons. A feasibility study of this type of hull must investigate the areas of transverse stability, resistance, reserve buoyancy, the hull form above the waterline and other features. For many purposes, this type of hull would not be satisfactory since the access for cargo handling is limited, the deck space for passengers is limited and the large draft would limit the movements of the vessel. For a tanker, however, many of these disadvantages are not as important as they are for other types. The cargo is handled by piping which reduces the need of access to the below deck spaces. Passengers are not carried and deck space is not essential for the crew. Even the large draft is not critical since tankers frequently load and unload outside of shallow harbors. A feasibility study of this type of hull form may prove to be worthwhile.

CONCLUSIONS

1. The formula, $Ck = 0.00655(f)(h)(n)$, is not accurate for vessels of very high displacement-length ratios and small actual lengths.

2. The formula, $Ck = 0.00655(f)(h)(n)$, is sufficiently accurate to be used to predict the effect of variations in hull form parameters on the natural pitching period.

3. The significant methods of reducing the $T_p/\sqrt{L}$ ratio at any displacement-length ratio are to increase the draft and the waterplane area. This implies that V-form sections are advantageous in decreasing the $T_p/\sqrt{L}$ ratio as well as to provide increased damping of motion.

4. The curve denoting the critical condition of operation (Fig. V) agrees very closely with Lewis' curve [7] at the lower displacement-length ratios, but shows a marked difference at high displacement-length ratios.

5. The $T_p/\sqrt{L}$ ratio can be markedly increased by decreasing the length and beam at the waterline in relation to the maximum length and beam of the submerged volume.

6. Large values of $T_p/\sqrt{L}$ make "supercritical" operation possible for ships with the speed-length ratios which are obtainable at the present.

RECOMMENDATIONS

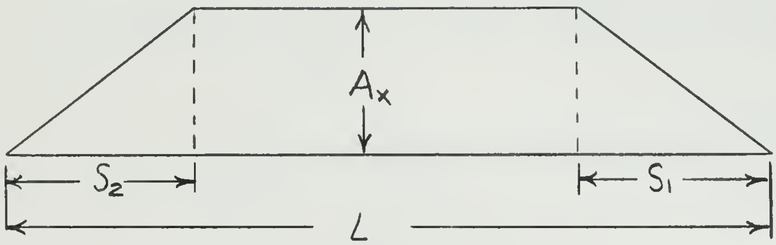
1. The effect of ship velocity on the coefficient of accession to inertia should be studied.
2. The effect of damping, coupling to heave, and large pitching angles on the natural pitching period should be studied.
3. Weight distribution data was available for only a few ships other than naval ships. The mass moment of inertia for a large variety of ships should be obtained to verify the approximations used for the constant c.
4. A greater number of ships should be studied at the high displacement-length ratios.
5. A study should be undertaken to determine the feasability of a hull form designed for supercritical operation. This type of hull may prove satisfactory for a tanker type ship.

APPENDIX

DETAILS OF PROCEDURE

Development of approximation for $\sqrt{\dfrac{k_v^{\,2}}{\alpha_L}}$

Assuming that the shape of the sectional area curve can be approximated as:



For $S_1 = S_2 = S$:

$$\nabla = A_x(L-S)$$

and $C_p = \dfrac{\nabla}{A_x L} = 1 - \dfrac{S}{L}$

then:

$$35\ I_{xs} = \frac{A_x}{12}\ (L - 2S)^3 + \frac{2A_x}{36}\ (S)^3 + A_x S\left(\frac{L}{2} - \frac{2}{3}S\right)^2$$

which reduces to

$$35\ I_{xs} = \frac{A_x}{12}\left(L^3 - 3L^2S + 4S^2L - 2S^3\right)$$

$$\frac{35\ I_{xs}}{A_x L^3} = \frac{1}{12}\left(1 - 3\left(\frac{S}{L}\right) + 4\left(\frac{S}{L}\right)^2 - 2\left(\frac{S}{L}\right)^3\right)$$

since $\dfrac{S}{L} = 1 - C_p$

$$\frac{35\ I_{xs}}{A_x L^3} = \frac{C_p}{12}\left(2\ C_p{}^2 - 2\ C_p + 1\right)$$

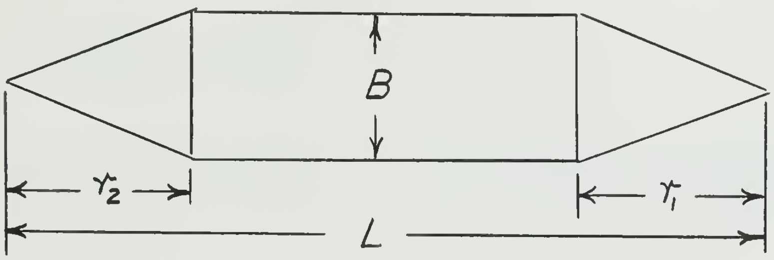
but $I_{xs} = \frac{\nabla}{35}\ K_{xs}{}^2 = \frac{C_p A_x L^3}{35}\ k_{xs}{}^2$

therefore

$$k_{xs}{}^2 = \frac{1}{12}\left(2\ C_p{}^2 - 2\ C_p + 1\right)$$

where k_{xs} is the gyradius about the midship section for a symmetrical hull form, divided by the length.

Assuming that the waterplane shape can be approximated as:



<u>For $r_1 = r_2 = r$:</u>

$$J_{xs} = \frac{B}{12}(L - 2r) + \frac{2B}{36}(r)^3 + B\ r\ \left(\frac{L}{2} - \frac{2}{3}\ r\right)^2$$

since $A_{WP} = B\ (L - r)$

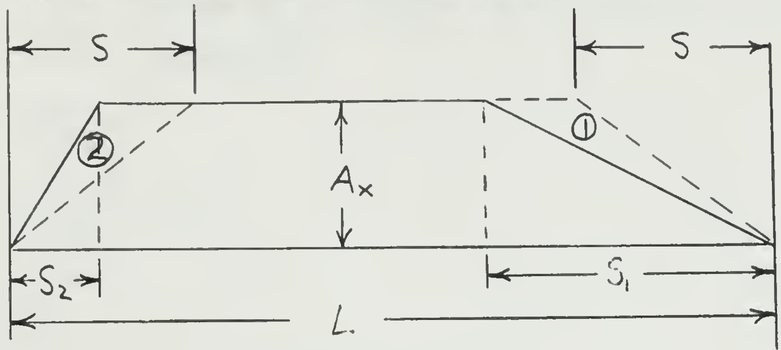
$$C_W = \frac{A_{WP}}{BL} = 1 - \frac{r}{L}$$

and $J_{xs} = \alpha_{xs}\ BL^3$

$$\frac{J_{xs}}{BL^3} = \alpha_{xs} = \frac{C_W}{12}\left(2C_W{}^2 - 2\ C_W + 1\right)$$

where α_{xs} is the longitudinal moment of inertia coefficient taken about the midship section, for a symmetrical waterplane shape.

For the more complicated, unsymmetrical case, it is assumed that the sectional area curve can be approximated as:



where $S_1 \neq S_2$ and S is defined as $S = \frac{S_1 + S_2}{2}$

$$\nabla = A_x \left(L - \frac{S_1}{2} - \frac{S_2}{2}\right) = A_x \left(L - S\right)$$

$$C_P = 1 - \frac{1}{2}\left(\frac{S_1}{L}\right) - \frac{1}{2}\left(\frac{S_2}{L}\right)$$

Defining the area of triangle 1 as A_1 and the area of triangle 2 as A_2

$$A_1 = A_2 = A$$

then

$$35\ I_x = 35\ I_{xs} + 35\ I_2 - 35\ I_1 + A_2\ (x_2 L)^2 - A_1 (x_1 L)^2$$

where

I_x is the mass moment of inertia of the unsymmetrical area about the midship section

I_2 and I_1 are the mass moments of inertia of the areas 2 and 1 about their own centroids.

I_{xs} is the mass moment of inertia of an equal, symmetrical area

x_2 and x_1 are the distances from the centroids of areas 2 and 1 to the midship section divided by the length.

Also assuming that

$$I_2 - I_1 \approx 0$$

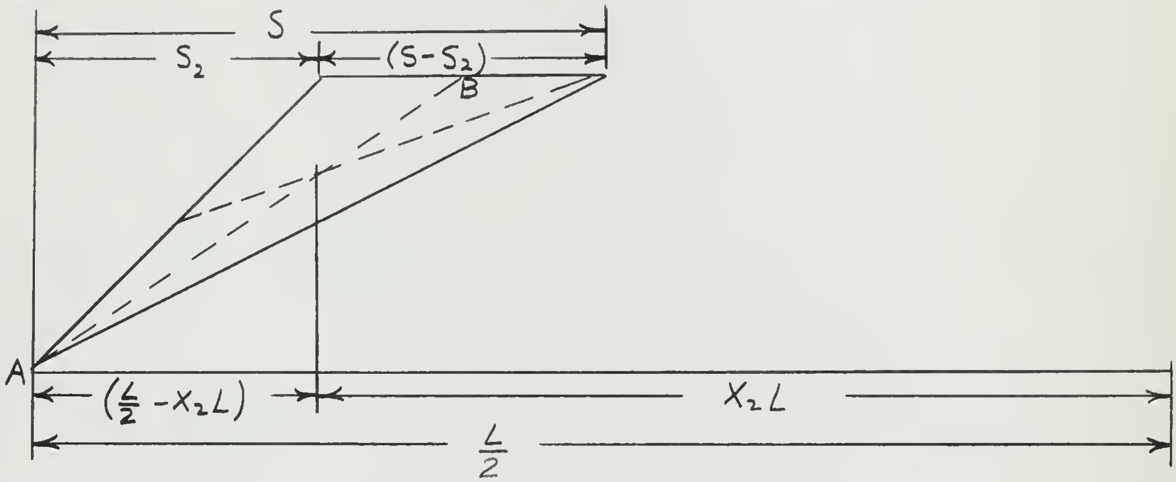
and $\quad d \nabla = A_2 x_2 + A_1 x_1$

where d is the distance from the longitudinal center of buoyancy to the midship section divided by the length. d is defined as negative when the LCB is aft of amidships.

Since $35 I = \nabla L^2 k^2$

$$k_x^2 = k_{xs}^2 + d\,(x_2 - x_1)$$

For area 2



by similar triangles

$$\frac{\frac{L}{2} - x_2 L}{\frac{2}{3}\,\overline{A\;B}} = \frac{S_2 + \frac{S-S_2}{2}}{\overline{A\;B}}$$

and $x_2 = \frac{1}{12}\left[6 - 2\left(\frac{S_1}{L}\right) - 6\left(\frac{S_2}{L}\right)\right]$

by symmetry

$$x_1 = \frac{1}{12}\left[6 - 6\left(\frac{S_1}{L}\right) - 2\left(\frac{S_2}{L}\right)\right]$$

$$(x_2 - x_1) = \frac{1}{3}\left[\frac{S_1}{L} - \frac{S_2}{L}\right]$$

$$(x_2 + x_1) = \frac{1}{12}\left[12 - 8\left(\frac{S_1}{L}\right) - 8\left(\frac{S_2}{L}\right)\right]$$

since $C_P = 1 - \frac{1}{2}\left(\frac{S_1}{L}\right) - \frac{1}{2}\left(\frac{S_2}{L}\right)$

$$16\ C_P - 4 = 12 - 8\left(\frac{S_1}{L}\right) - 8\left(\frac{S_2}{L}\right)$$

$$(x_2 + x_1) = \frac{1}{3}(4\ C_P - 1) = \frac{d\nabla}{A}$$

$$\frac{\nabla}{A} = \frac{L\ A_x C_P}{\frac{A_x}{4}(S_1 - S_2)}$$

and $\frac{1}{3}\left(\frac{S_1}{L} - \frac{S_2}{L}\right) = \frac{(4\ C_P)\ d}{4\ C_P - 1}$

therefore

$$(x_2 - x_1) = \frac{(4\ C_P)d}{4\ C_P - 1}$$

and

$$k_x^{\ 2} = k_{xs}^{\ 2} + \frac{(4\ C_P)d^2}{4\ C_P - 1}$$

by similarity, for an unsymmetrical waterplane:

$$\alpha_x = \alpha_{xs} + \frac{4\ C_W^{\ 2}\ \bar{x}^{\ 2}}{4\ C_W - 1}$$

where:

α_x is the longitudinal moment of inertia coefficient of an unsymmetrical waterplane about the midship section.

α_{xs} is the longitudinal moment of inertia coefficient of a symmetrical waterplane of equal area, about the midship section.

$\bar{x}$ is the distance from the longitudinal center of flotation to the midships section divided by the length.

$$k_v^2 = k_x^2 - d^2$$

$$\alpha_L = \alpha_x - C_w \ell^2$$

therefore:

$$\frac{k_v^2}{\alpha_L} = \frac{k_{xs}^2 + \dfrac{4\, C_P\, d^2}{4\, C_P - 1} - d^2}{\alpha_{xs} + \dfrac{4\, C_w^2 \ell^2}{4\, C_w - 1} - C_w \ell^2}$$

and

$$\frac{k_v^2}{\alpha_L} = \frac{k_{xs}^2 + \dfrac{d^2}{4\, C_P - 1}}{\alpha_{xs} + \dfrac{C_w \ell^2}{4\, C_w - 1}}$$

and approximately

$$\frac{k_v^2}{\alpha_L} \approx \frac{k_{xs}^2}{\alpha_{xs} + \dfrac{C_w \ell^2}{4\, C_w - 1} - \dfrac{d^2}{4\, C_P - 1}}$$

since $C_P \approx C_w$

$$\frac{k_v^2}{\alpha_L} \approx \frac{k_{xs}^2}{\alpha_{xs}}$$

when $C_w \ell^2 - d^2 \approx 0$

and therefore:

$$\left(\frac{k_v^2}{\alpha_L}\right)^{1/2} = \left[\frac{2\, C_P^2 - 2\, C_P + 1}{C_w (2\, C_w^2 - 2 C_w + 1)}\right]^{1/2}$$

Values of $C = \frac{I_y}{I_v}$

The following data represents the results obtained from computations of I_y and I_v for the twenty ships for which complete data was available. From these results, predictions for the value of C were made for the remaining vessels. The value of C used was determined as an average for the ships of similar type. For those types of vessels for which no weight data was available, a value of C = 1.0 was used.

TABLE II

VALUES OF C

Ship	C	Ship	C
Destroyers		Miscellaneous	
DD348	1.17	CB1	.956
DD356	1.14	CM5	1.03
DD445	1.11		
DD692 (S.H.)	1.155	Battleships	
DD692 (L.H.)	1.071	BB61	.946
DD931	1.45		
		Dry Cargo	
Cruisers		C-4	.993
CA32	1.26		
CA68	1.155	Tankers	
CA139	0.961	Hull No. 4476	.985
CL40	1.135		
CL51	0.962		
CL55	1.08		
CL144	1.07		
Aircraft Carriers			
CV32	1.39		
CVB42	1.49		

FIGURE VI

COEFFICIENT OF ACCESSION TO INERTIA

REPRODUCED FROM REF. [16]

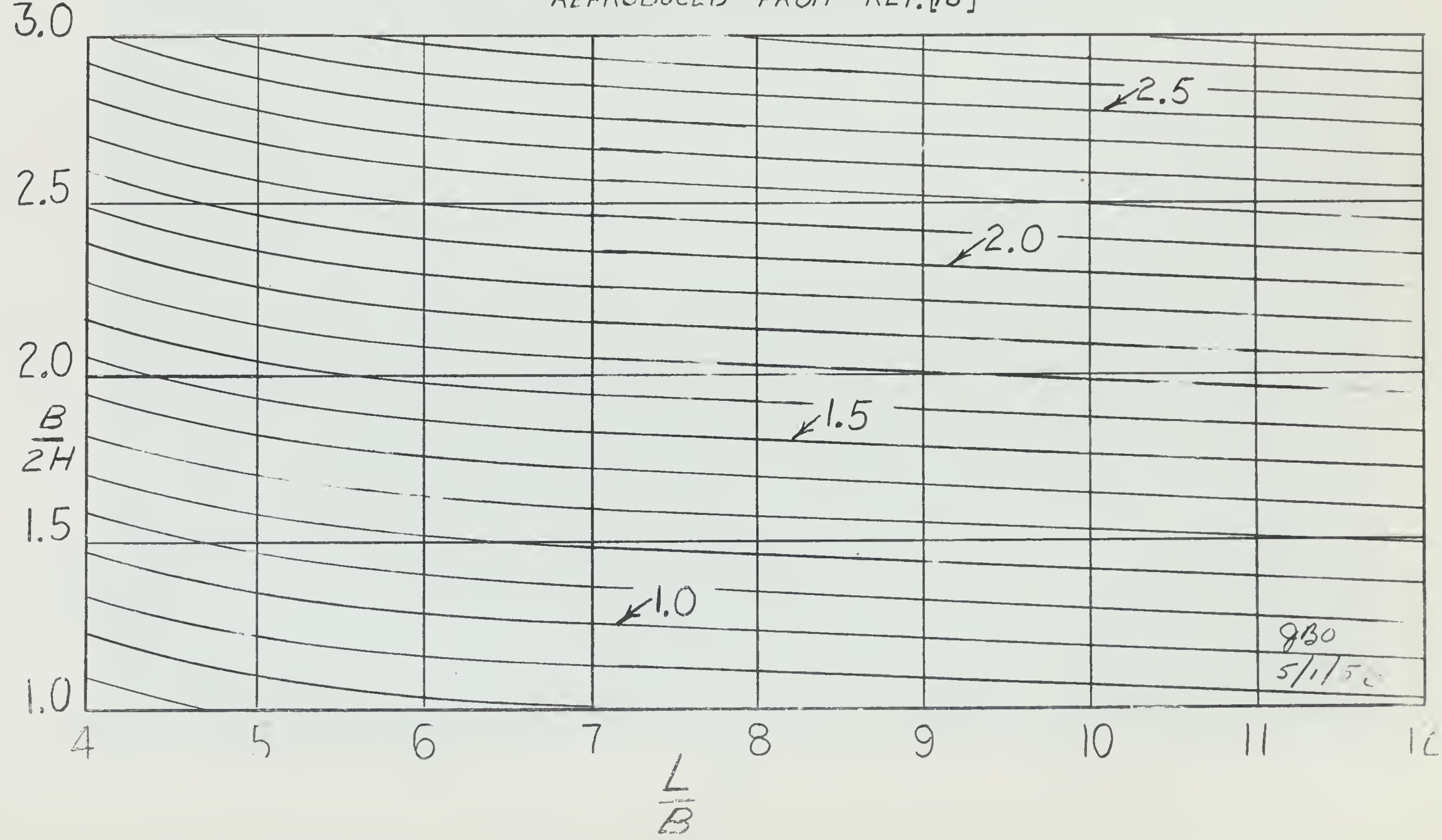


FIGURE VII

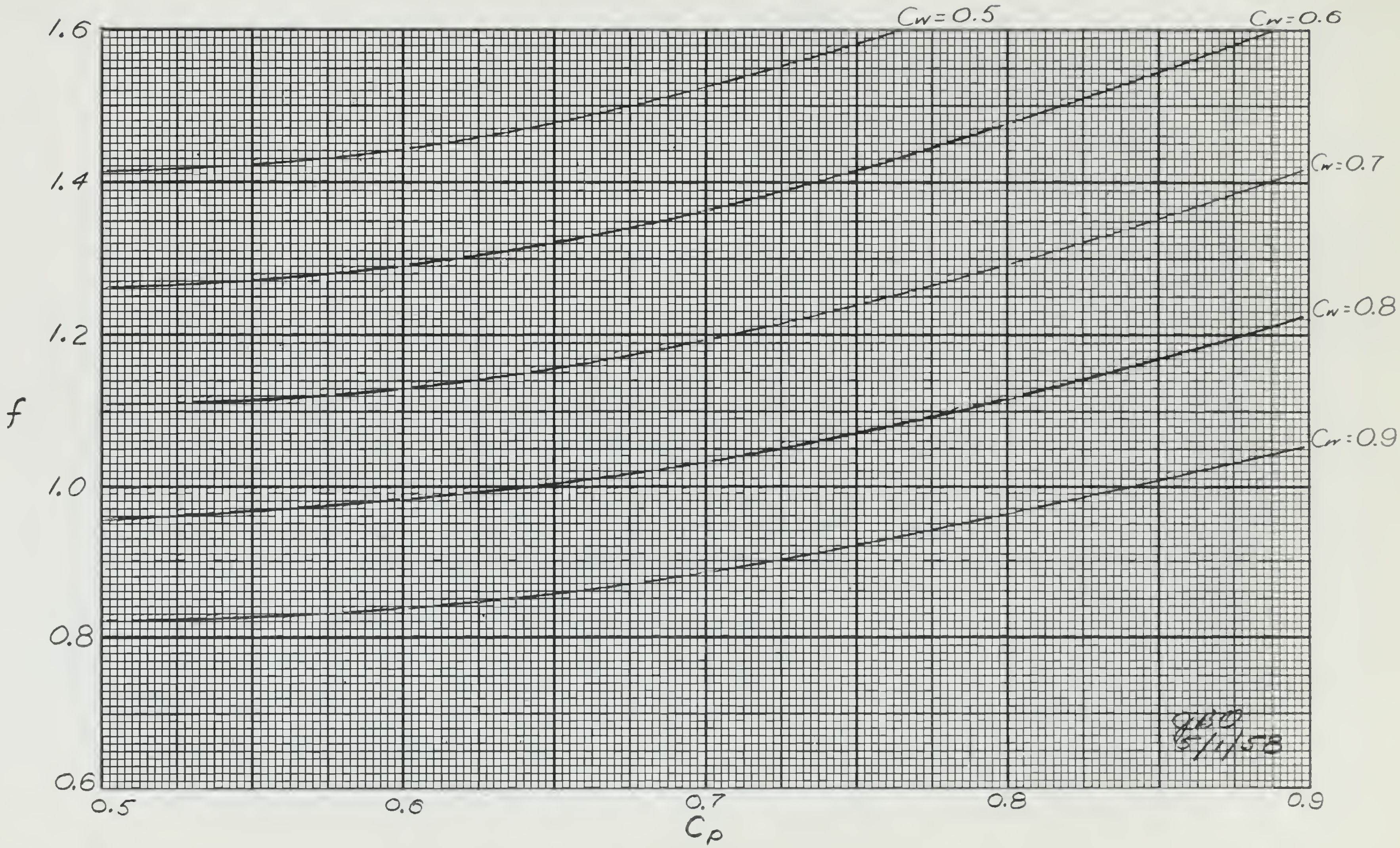


FIGURE VIII

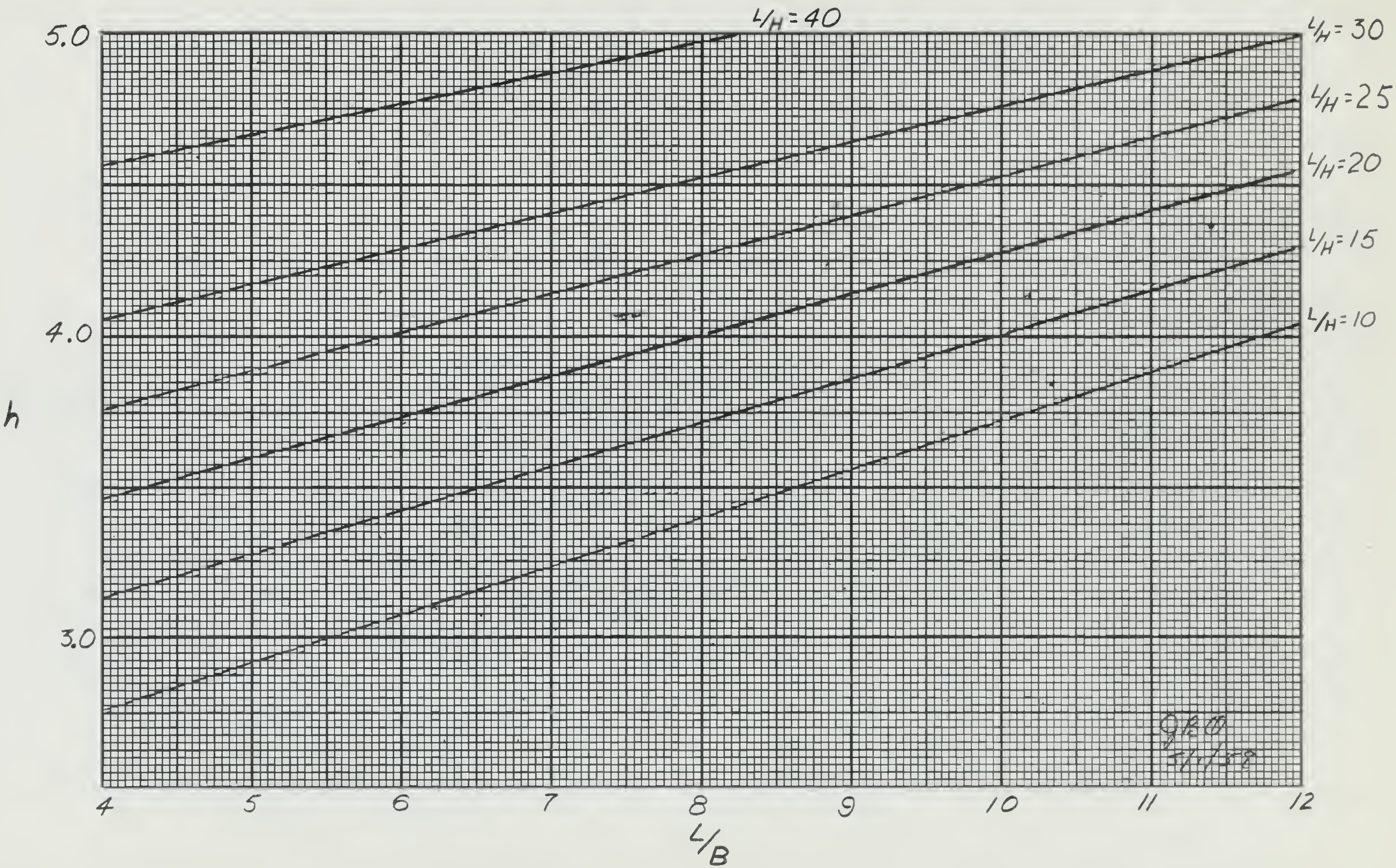
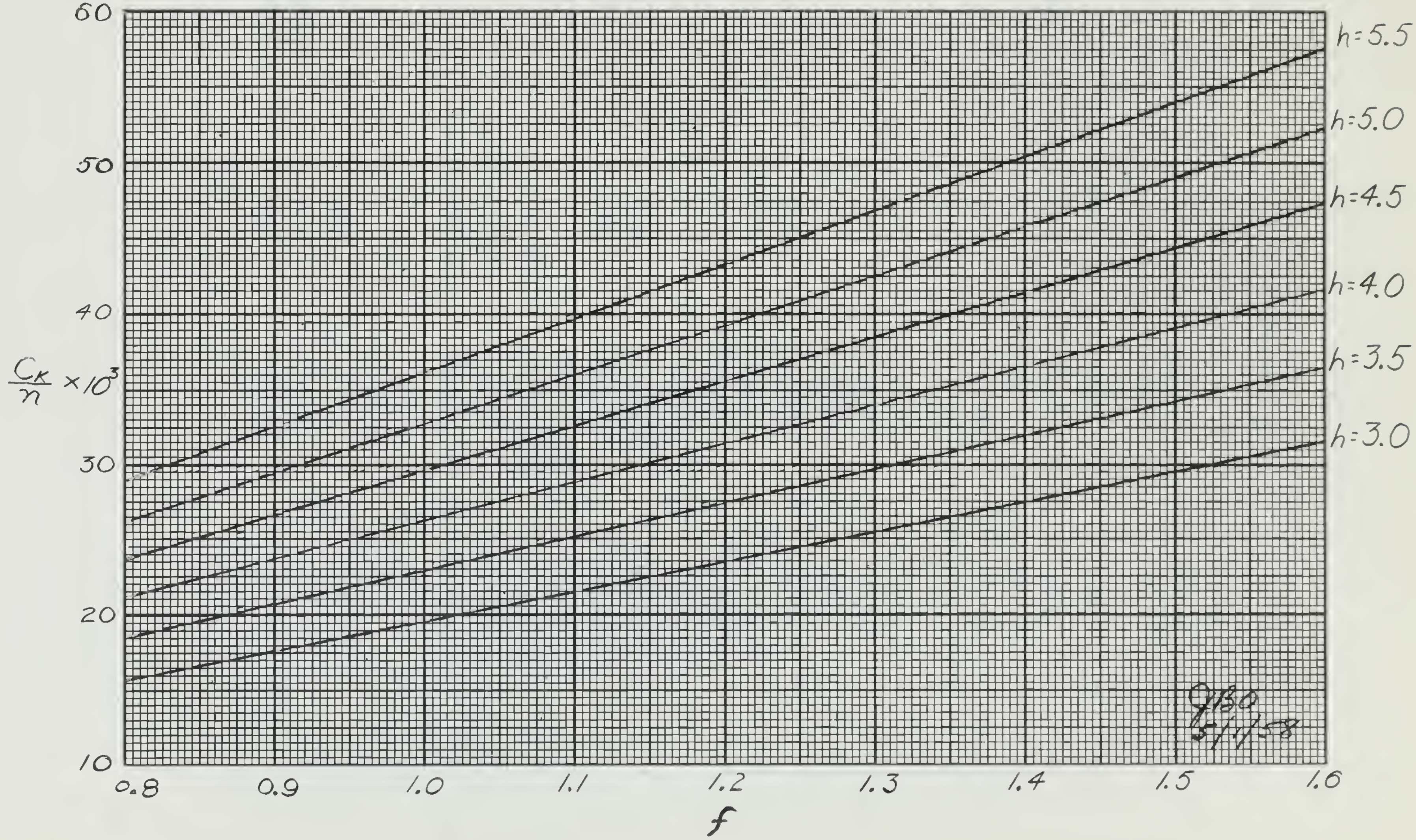


FIGURE IX



60
50
40
30
20
10
$\frac{C_K}{n} \times 10^3$
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
f
h=5.5
h=5.0
h=4.5
h=4.0
h=3.5
h=3.0

Table III

SUMMARY OF DATA

	No. 1	No. 2	No. 3	No. 4
Ship	CV-4 Ranger	DD-356	DD-348	CL-51
$\triangle$	15,742	2105	1725	7400
$\triangle/(.01L)^3$	40.5	40.9	46.3	49.7
L	730.	372.	334.	530
B	73.32	36.58	33.96	52.38
H	20.58	11.25	10.2	19.81
L/B	9.92	10.22	9.85	10.10
B/H	3.56	3.25	3.33	2.64
L/H	35.5	33.1	32.8	26.7
C_B	0.456	0.501	0.516	0.500
C_P	0.562	0.630	0.620	0.595
C_X	0.813	0.795	0.833	0.840
C_w	0.671	0.743	0.741	0.740
d	-0.0174	-0.01262	-0.01615	-0.01545
l	-0.0571	-0.03061	-0.05025	-0.0349
I_y	--	15,734,461	10,690,061	101,037,294
I_v	355,476,786	13,800,000	9,126,230	105,053,020
c	1.40 Assumed	1.140	1.170	0.963
J_y	1,029,000,000	80,600,000	51,600,000	314,979,000
k_{yy}	1.53	1.39	1.41	1.12
T_P	6.39	4.31	4.43	5.46
$T_P/\sqrt{L}$	0.2362	0.2235	0.2425	0.2375
C_K Computed	0.0371	0.0349	0.0357	0.0337
C_K Predicted	0.0390	0.0351	0.0348	0.0360
% Error	+5.11	+0.57	-2.51	+6.82

	No. 5	No. 6	No. 7	No. 8
Ship	DD 445	CV Yorktown	CA 68	CL 40
Δ	2550	23,600	15,153	11,450
$\Delta/(.01L)^3$	50.7	51.6	51.7	53.0
L	369	770	664	600
B	38.5	81.52	69.6	60.54
H	12.45	24.375	22.0	22.0
$L/_B$	9.60	9.44	9.55	9.92
$B/_H$	3.09	3.35	3.165	2.75
$L/_H$	29.6	31.6	30.2	27.2
C_B	0.503	0.526	0.523	0.515
C_P	0.630	0.549	0.614	0.613
C_X	0.799	0.959	0.852	0.840
C_w	0.756	0.723	0.744	0.685
d	-0.02115	-0.00965	-0.01175	-0.0697
l	--	--	-0.0695	-0.0725
I_y	18,977,052	--	349,853,440	220,168,903
I_v	17,091,344	555,894,590	302,513,768	194,090,040
c	1.110	1.40 Assumed	1.155	1.135
J_y	85,600,000	1,459,000,000	836,000,000	541,000,000
k_{yy}	1.30	1.47	1.33	1.16
T_P	4.53	6.82	6.20	5.94
$T_P/\sqrt{L}$	0.2360	0.2460	0.2405	0.2421
C_K Computed	0.0332	0.0343	0.0335	0.0333
C_K Predicted	0.0333	0.0340	0.0334	0.0329
% Error	+0.31	-0.88	-0.30	-1.20

	No. 9	No. 10	No. 11	No. 12
Ship	DD-692 (Long Hull)	CL-55	CA-139	DD-692 (Short Hull)
$\triangle$	3050	11,980	19,560	2880
$\triangle/(.01L)^3$	54.3	55.5	57.0	57.3
L	383	600	700	369.
B	40.58	64.96	74.0	40.2
H	13.12	21.75	24.0	13.0
$L/_B$	9.44	9.25	9.45	9.20
$B/_H$	3.09	2.99	3.08	3.09
$L/_H$	29.2	27.6	29.2	28.4
C_B	0.526	0.504	0.540	0.521
C_P	0.634	0.610	0.614	0.628
C_X	0.830	0.826	0.880	0.830
C_w	0.766	0.726	0.760	0.757
d	-0.01930	-0.012	-0.00628	-0.0200
l	-0.0584	-0.0681	-0.0675	-0.0607
I_y	24,199,375	213,239,213	414,553,319	20,969,821
I_v	22,576,160	197,479,000	430,563,487	18,152,334
c	1.071	1.080	0.962	1.153
J_y	101,500,000	560,000,000	1,073,100,000	88,600,000
k_{yy}	1.30	1.26	1.29	1.28
T_P	5.56	5.95	6.22	4.621
$T_P/\sqrt{L}$	0.2843	0.2430	0.2355	0.2405
C_K Computed	0.0329	0.0326	0.0312	0.0318
C_K Predicted	0.0320	0.0334	0.0322	0.0318
% Error	-2.18	+2.45	+3.12	0.0

	No. 13	No. 14	No. 15	No. 16
Ship	DD931	CA 32	CL-144	CV-32
$\triangle$	3900	11,440	17,350	33,500
$\triangle/(.01L)^3$	57.8	59.4	59.4	60.8
L	407.	578	664	820
B	44.96	60.64	68.6	93.80
H	14.5	21.5	24.0	26.0
$L/_B$	9.07	9.52	9.67	8.75
$B/_H$	3.12	2.83	2.86	3.61
$L/_H$	28.1	26.9	27.6	31.5
C_B	0.515	0.532	0.548	0.586
C_P	0.633	0.618	0.621	0.604
C_X	0.813	0.862	0.882	0.972
C_w	0.757	0.771	0.766	0.723
d	-0.0105	-0.0407	-0.01052	-0.00915
l	-0.0516	-0.00207	-0.0655	-0.0426
I_y	41,381,622	186,926,000	370,929,537	1,247,666,295
I_v	31,630,252	148,300,380	347,393,354	898,676,065
c	1.305	1.259	1.069	1.389
J_y	136,752,000	441,000,000	870,105,600	1,901,088,000
k_{yy}	1.29	1.19	1.21	1.50
T_P	5.07	5.95	6.229	7.66
$T_P/\sqrt{L}$	0.2520	0.2475	0.2418	0.2680
C_K Computed	0.0332	0.0321	0.0314	0.0344
C_K Predicted	0.0319	0.0316	0.0314	0.0342
% Error	-3.92	-1.51	0	-0.60

	No. 17	No. 18	No. 19	No. 20
Ship	CVL-48	CB-1	CV-2 Lexington	CVB-42
$\triangle$	17,800	31,725	43,500	54,600
$\triangle/(.01L)^3$	60.9	64.3	70.8	75.0
L	664.	790.	850.	900.
B	75.56	86.0	104.	113.
H	24.0	30.0	30.77	32.0
$L/_B$	8.78	9.19	8.17	7.97
$B/_H$	3.15	2.865	3.39	3.53
$L/_H$	27.7	26.3	27.6	28.1
C_B	0.512	0.545	0.551	0.587
C_P	0.619	0.591	0.564	0.605
C_X	0.829	0.922	0.980	0.971
C_w	0.741	0.717	0.639	0.718
d	-0.0182	-0.01345	+0.00186	-0.00755
l	-0.0723	-0.0592	--	-0.0480
I_y	--	842,995,240	--	2,529,130,968
I_v	379,217,650	880,810,363	1,251,145,588	1,700,839,205
c	1.40 Assumed	0.956	1.40 Assumed	1.489
J_y	913,332,000	1,702,134,000	1,898,000,000	3,099,600,000
k_{yy}	1.30	1.20	1.43	1.46
T_P	6.94	7.31	8.91	8.31
$T_P/\sqrt{L}$	0.2590	0.2602	0.3040	0.2770
C_K Computed	0.0332	0.0325	0.0362	0.0320
C_K Predicted	0.0317	0.0330	0.0358	0.0338
% Error	-4.51	+1.51	-1.10	+5.61

	No. 21	No. 22	No. 23	No. 24
Ship	Cargo, Ore Sname No. 129	Passenger Sname No. 33	PG-50	Barge, Car Sname No. 148
$\triangle$	15,860	68,324	2420	2010
$\triangle/(.01L)^3$	75.04	79.5	82.9	83.91
L	595.67	950	308	285
B	51.20	100	43.9	44.3
H	20.50	40.0	13.0	10.0
$L/_B$	11.634	9.29	7.02	6.434
$B/_H$	2.497	2.50	3.375	4.429
$L/_H$	29.06	23.75	23.7	28.5
C_B	0.887	0.632	0.482	0.539
C_P	0.890	0.651	0.576	0.721
C_X	0.997	0.970	0.836	0.747
C_W	0.925	0.762	0.680	0.817
d	+0.006	-0.001	-0.00162	-0.105
l	-0.002	-0.011	0	-0.053
I_y	--	--	--	--
I_v	283,768,820	3,037,536,330	9,544,427	8,398,033
c	1.00 Assumed	1.00 Assumed	1.10 Assumed	1.00 Assumed
J_y	591,508,960	3,654,166,000	45,250,000	54,602,236
k_{yy}	1.09	1.03	1.48	1.85
T_P	6.559	8.515	4.83	4.33
$T_P/\sqrt{L}$	0.2688	0.2765	0.2755	0.2570
C_K Computed	0.0310	0.0310	0.0303	0.0281
C_K Predicted	0.0322	0.0308	0.0305	0.0289
% Error	+3.87	-0.64	+0.66	+2.85

	No. 25	No. 26	No. 27	No. 28
Ship	BB-61	LST-1153	Cargo, Orr Sname No. 130	CM-5
Δ	54,400	4600	25,430	7900
$\Delta/(.01L)^3$	85.6	92.3	93.78	97.0
L	860.	368.	647.25	440.
B	108.2	54.0	67.0	60.0
H	34.5	10.0	24.0	18.0
$L/_B$	7.95	6.81	9.66	7.33
$B/_H$	3.14	5.40	2.792	3.33
$L/_H$	24.9	36.8	27.0	24.45
C_B	0.592	0.811	0.855	0.581
C_P	0.595	0.811	0.861	0.604
C_X	0.996	0.9999	0.993	0.964
C_W	0.696	0.887	0.902	0.733
d	-0.024	-0.014	+0.016	-0.00682
l	--	-0.00136	+0.007	-0.0455
I_y	1,652,943,200	--	--	67,936,384
I_v	1,750,042,560	38,621,496	447,869,210	65,898,218
c	0.945	1.00 Assumed	1.00 Assumed	1.015
J_y	2,394,033,600	162,288,000	820,906,570	197,736,000
k_{yy}	1.29	2.32	1.16	1.36
T_P	8.37	5.823	7.09	5.84
$T_P/\sqrt{L}$	0.3059	0.3041	0.2790	0.2785
C_K Computed	0.0323	0.0317	0.0281	0.0293
C_K Predicted	0.0331	0.0306	0.0289	0.0300
% Error	+2.48	-3.47	+2.85	+2.38

	No. 29	No. 30	No. 31	No. 32
Ship	Passenger Sname No.16	Passenger Sname No.21	Passenger Sname No.28	Passenger Mariposa
$\triangle$	19,900	34957	35,926	25,010
$\triangle/(0.1L)^3$	97.0	99.0	100.38	101
L	590	707	710	628
B	75.5	93.0	96.0	79.0
H	25.0	32.25	32.0	27.0
$L/_B$	7.81	7.60	7.396	7.95
$B/_H$	3.02	2.89	3.000	2.92
$L/_H$	23.60	21.92	22.19	23.21
C_B	0.617	0.577	0.5765	0.647
C_P	0.650	0.590	0.5895	0.653
C_X	0.949	0.979	0.978	0.990
C_w	0.741	0.727	0.7146	0.745
d	+0.003	0.0	-0.005	-0.00038
l	-0.030	-0.0267	-0.0363	--
I_y	--	--	--	--
I_v	314,799,328	727,434,724	755,510,002	441,145,804
c	1.00 Assumed	1.00 Assumed	1.00 Assumed	1.10 Assumed
J_y	602,841,800	1,235,089,186	1,262,956,237	776,100,000
k_{yy}	1.24	1.18	1.22	1.19
T_P	7.07	7.434	7.552	7.42
$T_P/\sqrt{L}$	0.2910	0.2795	0.2834	0.2890
C_K Computed	0.0296	0.0281	0.0283	0.0289
C_K Predicted	0.0292	0.0287	0.0292	0.0300
% Error	-1.37	+2.14	+3.90	+3.80

Ship	No. 33 Passenger Sname No.32	No. 34 Cargo, Ore Sname No.128	No. 35 Ferry, Double Ended Sname No.150	No. 36 LST-1
Δ	6726.	12,630.	844.	3610
$\Delta/(.01L)^3$	105.5	108.9	109.7	114.0
L	400.	487.67	197.44	31.6
B	61.0	51.20	38.0	50.0
H	18.0	20.50	10.0	10.0
$L/_B$	6.557	9.526	5.196	6.32
$B/_H$	3.389	2.497	3.800	5.00
$L/_H$	22.22	23.79	19.74	31.6
C_B	0.536	0.862	0.395	0.805
C_P	0.566	0.865	0.589	0.806
C_X	0.947	0.997	0.670	0.998
C_w	0.701	0.908	0.716	0.895
d	-0.0083	+0.009	0.0	-0.0142
l	-0.036	-0.001	0.0	0.0
I_y	--	--	--	--
I_v	42,750,585	151,312,667	1,270,534	21,873,670
c	1.00 Assumed	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	135,595,354	318,170,718	9,883,843	96,222,000
k_{yy}	1.37	1.04	1.49	2.09
T_P	5.659	6.452	3.705	5.48
$T_P/\sqrt{L}$	0.2830	0.2919	0.2640	0.3080
C_K Computed	0.0275	0.0279	0.0252	0.0289
C_K Predcited	0.0268	0.0289	0.0265	0.0284
% Error	-2.55	+3.60	+5.16	-1.73

	No. 37	No. 38	No. 39	No. 40
Ship	LSM-1	BB-55	Passenger Sname No.37	AD-32
Δ	875	42,061	13,666	16,800
$\Delta/(0.1L)^3$	115.1	115.8	118.8	119.5
L	196.5	714	486.2	520
B	34.0	104	56.0	73
H	6.00	31.5	24.0	24
$L/_B$	5.78	6.85	8.679	7.13
$B/_H$	5.67	3.30	2.338	3.04
$L/_H$	32.75	22.67	20.26	21.65
C_B	0.764	0.623	0.732	0.645
C_P	0.767	0.623	0.737	0.655
C_X	0.995	1.000	0.992	0.985
C_w	0.900	0.705	0.801	0.750
d	-0.0272	-0.01675	-0.0192	-0.00549
l	-0.0206	--	-0.028	--
I_y	--	--	--	218,657,700
I_v	15,759,987	898,173,612	159,313,663	187,432,560
c	1.00 Assumed	0.945 Assumed	1.00 Assumed	1.165
J_y	16,500,000	1,415,000,000	299,484,384	406,879,200
k_{yy}	2.42	1.35	0.94	1.23
T_P	4.146	7.906	6.655	6.87
$T_P/\sqrt{L}$	0.2955	0.2956	0.3018	0.3015
C_K Computed	0.0276	0.0274	0.0277	0.0276
C_K Predicted	0.0272	0.0292	0.0281	0.0292
% Error	-1.45	+6.57	+1.44	+5.79

Ship	No. 41 Passenger Sname No.47	No. 42 Launch, Survey Sname No.143	No. 43 Cargo C-4	No. 44 Passenger and Freight Sname No.49
$\triangle$	8518.	75.85	18,610	28,353.
$\triangle /(.01L)^3$	125.125	125.51	126.3	128.69
L	408.	84.57	528.	607
B	62.0	16.31	76.	80.0
H	20.33	4.52	27.	30.0
$L/_B$	6.581	5.186	6.95	7.5328
$B/_H$	3.051	3.106	2.815	2.6535
$L/_H$	20.07	18.70	19.55	20.23
C_B	0.5797	0.365	0.6125	0.6812
C_P	0.5988	0.613	0.6246	0.7050
C_X	0.9682	0.595	0.9807	0.9663
C_w	0.7234	0.720	0.7236	0.7738
d	-0.0013	-0.011	-0.0149	+0.0041
l	-0.033	-0.037	-0.0349	-0.0115
I_y	--	--	232,347,357	--
I_v	60,048,458	24,051.	233,936,260	511,124,401
c	1.00 Assumed	1.00 Assumed	0.99	1.00 Assumed
J_y	156,613,571	391,410.	393,845,760	762,449,107
k_{yy}	1.22	1.19	1.12	1.05
T_P	6.042	2.404	7.18	7.677
$T_P/\sqrt{L}$	0.2991	0.2613	0.3130	0.3121
C_K Computed	0.0267	0.0233	0.0278	0.0274
C_K Predicted	0.0274	0.0260	0.0280	0.0278
% Error	+2.62	+11.0	+0.72	+1.46

Ship	No. 45 AE-21	No. 46 Cargo Sname No.6	No. 47 Tanker Standard Oil	No. 48 BB Pennsylvania
$\triangle$	15,700	12,350	22,907	31,400
$\triangle/(.01L)^3$	136	137.5	142	145.3
L	487	448	544	600
B	72.0	61.0	75.0	96.67
H	26.5	26.5	29.83	28.7
$L/_B$	6.76	7.35	7.25	6.21
$B/_H$	2.72	2.30	2.51	3.37
$L/_H$	18.4	16.9	18.20	20.9
C_B	0.592	0.598	0.684	0.650
C_P	0.601	0.625	0.696	0.666
C_X	0.985	0.956	0.982	0.975
C_W	0.720	0.743	0.772	0.724
d	-0.065	+0.008	-0.010	-0.010
l	-0.048	-0.028	--	-0.020
I_y	--	--	--	--
I_v	156,553,955	106,606,838	317,429,470	504,246,776
c	1.00 Assumed	1.00 Assumed	0.985 Assumed	0.945 Assumed
J_y	294,537,600	221,785,295	533,240,000	782,900,000
k_{yy}	1.08	0.91	1.01	1.34
T_P	6.88	6.26	7.14	7.93
$T_P/\sqrt{L}$	0.3120	0.2960	0.3031	0.3240
C_K Computed	0.0270	0.0253	0.0255	0.0269
C_K Predicted	0.0268	0.0263	0.0270	0.0278
% Error	-0.74	+3.96	+5.87	+2.97

Ship	No. 49 Cargo Sname No.69	No. 50 BB-43	No. 51 Ferry, Car Sname No.100	No. 52 Cargo Sname No.87
$\triangle$	3592	32,550.	6775	21,423
$\triangle/(.01L)^3$	147.7	150.2	150.2	150.6
L	290.	600.	356.6	522.
B	43.0	97.40	67.86	70.0
H	16.5	30.5	16.5	27.0
$L/_B$	6.74	6.16	5.255	7.463
$B/_H$	2.60	3.19	4.112	2.592
$L/_H$	17.58	19.65	21.61	19.33
C_B	0.611	0.625	0.593	0.760
C_P	0.643	0.652	0.685	0.774
C_X	0.950	0.959	0.866	0.982
C_w	0.746	0.710	0.805	0.860
d	-0.0075	+0.00125	0.0	+0.031
l	--	-0.0222	0.0	-0.005
I_y	--	--	--	--
I_v	13,452,267	521,040,367	40,417,752	335,721,300
c	1.00 Assumed	0.945 Assumed	1.00 Assumed	1.00 Assumed
J_y	41,546,033	793,800,000	205,233,440	584,349,993
k_{yy}	1.02	1.27	1.62	1.03
T_P	5.299	7.87	4.70	7.09
$T_P/\sqrt{L}$	0.3108	0.3660	0.2490	0.3103
C_K Computed	0.0256	0.0299	0.0235	0.0252
C_K Predicted	0.0263	0.0285	0.0250	0.0257
% Error	+2.73	-5.01	+6.38	+1.98

	No. 53	No. 54	No. 55	No. 56
Ship	BB Delaware	Motor Sailer Sname 149	Tanker Sname 115	Tanker Sname 53
$\triangle$	20,000	153	4165	33,973
$\triangle/(.01L)^3$	151	153.4	154.3	157.1
L	510	100	300	600
B	84.84	22.83	49.0	82.5
H	26.95	9.08	14.9	31.4
$L/_B$	6.01	4.380	6.073	7.273
$B/_H$	3.15	2.514	3.312	2.629
$L/_H$	18.9	11.0	20.1	19.11
C_B	0.595	0.256	0.660	0.765
C_P	0.608	0.575	0.669	0.769
C_X	0.979	0.445	0.986	0.994
C_w	0.687	0.677	0.759	0.830
d	-0.01665	-0.028	+0.006	+0.0138
l	-0.0259	-0.030	-0.001	-0.0049
I_y	--	--	--	--
I_v	252,820,505	59,836	16,825,505	670,098,311
c	0.945 Assumed	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	404,830,000	670,385	54,737,000	896,911,362
k_{yy}	1.22	0.88	1.32	1.05
T_P	7.79	2.686	5.535	8.122
$T_P/\sqrt{L}$	0.3450	0.2686	0.3196	0.3315
C_K Computed	0.0273	0.0217	0.0257	0.0264
C_K Predicted	0.0281	0.0222	0.0262	0.0263
% Error	+2.93	+2.31	+1.94	-0.38

	No. 57	No. 58	No. 59	No. 60
Ship	Tanker Sname No.56	Cargo Sname No.17	Cargo C-2	Tanker Sname No.95
$\triangle$	32,620	19,825	13,841	23,066
$\triangle/(0.1L)^3$	159.0	163	164.1	165.8
L	590.	496	438.5	519.
B	82.5	71.5	63.0	72.82
H	32.0	30.0	25.75	30.0
$L/_B$	7.152	6.94	6.96	7.128
$B/_H$	2.577	2.38	2.45	2.425
$L/_H$	18.44	16.53	17.1	17.30
C_B	0.733	0.654	0.670	0.712
C_P	0.749	0.665	0.694	0.718
C_X	0.979	0.9848	0.980	0.992
C_W	0.810	0.752	0.753	0.793
d	+0.0024	+0.006	+0.01159	-0.0082
l	-0.0106	-0.0172	-0.0068	-0.0213
I_y	--	--	--	--
I_v	553,397,382	219,477,841	127,105,064	307,841,696
c	1.00 Assumed	1.00 Assumed	0.99 Assumed	1.00 Assumed
J_y	744,158,193	342,248,555	222,014,000	464,625,770
k_{yy}	1.02	0.94	0.99	0.97
T_P	8.043	7.32	6.88	7.480
$T_P/\sqrt{L}$	0.3310	0.3284	0.3340	0.3288
C_K Computed	0.0262	0.0257	0.0261	0.0255
C_K Predicted	0.0263	0.0265	0.0272	0.0255
% Error	+0.38	+3.11	+3.45	0.0

	No. 61	No. 62	No. 63	No. 64
Ship	Tanker T-2	Tanker Beth-Quincy	Cargo, Ore Sname 9	Cargo Sname 20
$\triangle$	22,893	36,044	8,844	14,820
$\triangle/(.01L)^3$	169.8	171	175	178
L	513.4	595	370.0	436.5
B	70.0	84	64.0	62.0
H	30.58	33	17.5	28.1
$L/_B$	7.34	7.09	5.78	7.04
$B/_H$	2.29	2.55	3.66	2.21
$L/_H$	16.80	18.0	21.14	15.55
C_B	0.749	0.7649	0.747	0.682
C_P	0.757	0.7699	0.753	0.691
C_X	0.989	0.9935	0.9922	0.988
C_w	0.796	0.8468	0.841	0.791
d	+0.02235	+0.0158	+0.022	+0.001
l	--	--	-0.010	-0.0038
I_y	--	686,805,000	--	--
I_v	312,587,749	679,037,000	65,805,040	127,182,631
c	0.985 Assumed	0.985	1.00 Assumed	1.00 Assumed
J_y	467,000,000	949,570,000	175,885,034	221,336,117
k_{yy}	0.93	1.02	1.46	0.88
T_P	7.42	7.88	6.281	6.819
$T_P/\sqrt{L}$	0.3278	0.3215	0.3259	0.3263
C_K Computed	0.0252	0.0247	0.0246	0.0244
C_K Predicted	0.0265	0.0250	0.0250	0.0245
% Error	+5.16	+1.42	+1.63	+0.40

	No. 65	No. 66	No. 67	No. 68
Ship	Cargo Sname No.67	Cargo Sname No.68	Tug, Fleet Sname No.80	Cargo Sname No.1
$\triangle$	10,585	13,900	149.	12,812
$\triangle/(.01L)^3$	185.6	191.6	200.1	208
L	385.	416.	195.	395
B	54.0	56.9	39.	60.0
H	24.75	27.15	14.1	27.5
$L/_B$	7.13	7.306	5.078	6.58
$B/_H$	2.181	2.097	2.714	2.18
$L/_H$	15.55	15.32	13.82	14.36
C_B	0.720	0.750	0.514	0.688
C_P	0.730	0.760	0.569	0.695
C_X	0.986	0.987	0.904	0.9896
C_w	0.811	0.860	0.661	0.785
d	+0.008	+0.021	-0.024	+0.0174
l	-0.0197	+0.0064	--	-0.0117
I_y	--	--	--	--
I_v	80,324,858	131,425,140	2,381,522	98,714,444
c	1.00 Assumed	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	149,199,832	228,743,915	8,750,800	167,931,326
k_{yy}	0.87	0.85	1.02	0.86
T_P	6.55	6.760	4.85	6.851
$T_P/\sqrt{L}$	0.3376	0.3314	0.3475	0.3443
C_K Computed	0.0248	0.0241	0.0245	0.0238
C_K Predicted	0.0252	0.0243	0.0252	0.0238
% Error	+1.60	+0.80	+2.85	0.0

Ship	No. 69 Tanker Sname 65	No. 70 Trawler Sname 125	No. 71 Dredge,Harbor Sname 103	No. 72 Trawler Sname 123
$\triangle$	2,217	706	3624	352.0
$\triangle/(.01L)^3$	213.4	234.5	241.25	270.2
L	212.5	144.4	246.7	110.49
B	37.0	27.1	46.0	22.48
H	13.0	12.8	14.92	10.69
$L/_B$	5.744	5.33	5.363	4.916
$B/_H$	2.842	2.115	3.082	2.102
$L/_H$	16.35	11.30	16.53	10.34
C_B	0.759	0.493	0.747	0.464
C_P	0.789	0.615	0.757	0.579
C_X	0.962	0.801	0.987	0.801
C_w	0.902	0.757	0.842	0.742
d	+0.0110	+0.008	+0.011	-0.009
l	-0.0165	-0.019	-0.010	-0.013
I_y	--	--	--	--
I_v	5,892,512	629,228	11,558,621	171,498
c	1.00 Assumed	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	22,333,004	3,431,341	36,357,972	1,241,394
k_{yy}	1.10	0.78	1.19	0.76
T_P	4.880	3.740	5.463	3.229
$T_P/\sqrt{L}$	0.3342	0.3114	0.3475	0.3069
C_K Computed	0.0219	0.0204	0.0223	0.0187
C_K Predicted	0.0219	0.0215	0.0231	0.0205
% Error	0.0	+5.40	+3.59	+9.63

Ship	No. 73 Trawler Sname No.111	No. 74 Tug,Ocean Going Sname No.86	No. 75 Tug,Ocean Going Sname No.75	No. 76 Tug, Ocean Going Sname No.94
$\triangle$	613.	1094.	738.5	517.8
$\triangle/(.01L)^3$	279.2	294.	301.3	347.
L	130.	155.0	134.8	114.58
B	28.5	34.7	33.8	30.1
H	12.5	12.8	12.36	11.3
$L/_B$	4.562	4.470	3.992	3.805
$B/_H$	2.280	2.710	2.733	2.665
$L/_H$	10.40	12.11	10.91	10.14
C_B	0.460	0.556	0.459	0.465
C_P	0.570	0.677	0.607	0.586
C_X	0.807	0.821	0.757	0.794
C_w	0.699	0.776	0.738	0.717
d	+0.001	-0.002	-0.002	-0.004
l	-0.036	-0.0252	-0.0223	-0.0221
I_y	--	--	--	--
I_v	407,229.	1,266,469	566,220	277,483
c	1.00 Assumed	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	2,247,498	5,614,318	3,393,202	1,748,920
k_{yy}	0.80	0.95	0.91	0.85
T_P	3.745	4.343	3.694	3.550
$T_P/\sqrt{L}$	0.3279	0.3486	0.3179	0.3318
C_K Computed	0.0196	0.0203	0.0183	0.0178
C_K Predicted	0.0203	0.0207	0.0194	0.0196
% Error	+3.57	+1.97	+6.01	+10.1

	No. 77	No. 78	No. 79
Ship	Tug, Harbor Sname 70	Tug Sname No. 72	Tug, Harbor Sname 73
$\triangle$	27.6	158.7	347.3
$\triangle/(01L)^3$	368	392.2	481.5
L	42.1	74.33	87.67
B	12.5	20.48	23.6
H	4.01	8.14	10.8
$L/_B$	3.365	3.623	3.720
$B/_H$	3.120	2.517	2.340
$L/_H$	10.50	9.27	8.118
C_B	0.458	0.453	0.544
C_P	0.595	0.589	0.623
C_X	0.770	0.770	0.873
C_W	0.757	0.742	0.776
d	-0.020	-0.021	-0.009
l	-0.045	--	-0.034
I_y	--	--	--
I_v	2061.3	32,448	119,791
c	1.00 Assumed	1.00 Assumed	1.00 Assumed
J_y	40,591.3	309,286	722,316
k_{yy}	0.98	0.98	0.70
T_P	2.083	2.95	3.475
$T_P/\sqrt{L}$	0.3210	0.3422	0.3717
C_K Computed	0.0167	0.0174	0.0169
C_K Predicted	0.0190	0.0180	0.0180
% Error	+13.8	+3.45	+6.5

Ships identified by "Sname" number are from Model and Expanded Resistance Data Sheets No. 1 - 150 of the Society of Naval Architects and Marine Engineers.

Table IV

Ship	(600x60) Ellipsoid E-1	(600x60) Ellipsoid E-2	(600x60) Ellipsoid E-3
$\triangle$	16154	20,920	25,196
$\triangle/(.01L)^3$	74.8	96.9	116.6
L_w	600	583.5	546
B_w	60	58.5	54.6
H	30.	36.	42.
$L/_B$	10.00	10	10
$B/_H$	2.00	1.667	1.429
$L/_H$	20	16.67	14.29
C_B	0.5235	0.565	0.583
C_P	0.665	0.691	0.697
C_X	0.7854	0.8171	0.8365
C_w	0.7854	0.7854	0.7854
d	0	0	0
l	0	0	0
I_y	--	--	--
I_v	293,679,720	395,019,430	475,787,159
c	1.0	1.0	1.0
J_y	636,336,000	570,641,182	436,378,872
k_{yy}	0.82 Assumed	0.68 Assumed	0.58 Assumed
T_P	6.003	7.075	8.613
$T_P/\sqrt{L}$	.2450	.2888	.3516
C_K Computed	.0284	.0293	.0326
C_K Predicted	.0283	.0295	.0330
% Error	-0.35	+0.68	+1.23

Table IV (Cont'd.)

Ship	(600x60) Ellipsoid E-4	(600x60) Ellipsoid E-5	(600x60) Ellipsoid E-6
$\triangle$	28,930	31,421	31,857
$\triangle/(.01L)^3$	134.0	145.5	147.5
L_w	480	360	300
B_w	48.	36.	30.
H	48.	54.	56.
$L/_B$	10	10	10
$B/_H$	1.25	1.11	1.071
$L/_H$	12.50	11.11	10.71
C_B	0.586	0.566	0.553
C_P	0.696	0.685	0.676
C_X	0.8420	0.8265	0.7854
C_w	0.7854	0.7854	0.8182
d	0	0	0
l	0	0	0
I_y	-	-	-
I_v	545,962,365	576,491,213	579,025,684
c	1.0	1.0	1.0
J_y	260,643,226	82,469,146	39,771,000
k_{yy}	0.49 Assumed	0.44 Assumed	0.42 Assumed
T_P	11.59	20.76	29.80
$T_P/\sqrt{L}$	.4731	.8473	1.216
C_K Computed	.0409	.0701	0.100
C_K Predicted	.0413	.0723	0.102
% Error	+0.98	+3.14	+2.00

C - SAMPLE CALCULATION
DATA SHEET

SHIP- BB61

Δ_{DWL}	54,400	L/H	24.9	C_B	.592
L	860.	B/H	3.14	C_P	.595
B	108.2	L/B	7.95	C_W	.096
$H_{\text{⊄}}$	34.5	MTI	6628	C_X	.996
$\Delta/(.01L)^3$	85.6	LCF		LCB	-20.7
		ℓ		d	-.024

$I_y =$ 1,652,943,200

$I_v =$ 1,750,042,560

$J_y =$ 2,394,033,600

$k_{yy} =$ 1.29

$$T_p = \frac{2\pi}{\sqrt{\rho g}} \sqrt{\frac{I_y + k_{yy} I_v}{J_y}}$$

$T_p =$ 8.37

$T_p/\sqrt{L} =$ .3059

$C_{K\,COMP} =$ .0323

$C_{K\,PRE} =$ .0331

ERROR = +2.48%

COMPUTATION SHEET

SHIP BB61
COMPUTATION OF Δ, I_V
$S = L/20 =$ ______

SOURCE OF DATA ____
CONTRACT PLANS

STA	AREA OR BEAM	SM	f(A)	ARM	f(M)	ARM	f(I)
0	0	1/4	0	10	0	10	0
1	539	1	539	9	4851	9	43659
2	825	3/4	619	8	4952	8	39616
4	1520	2	3040	6	18240	6	109440
6	2468	1	2468	4	9872	4	39488
8	3215	2	6430	2	12860	2	25720
10	3702	1	3702	0	50775	0	0
12	3718	2	7436	-2	14872	-2	29744
14	3395	1	3395	-4	13580	-4	54320
16	2533	2	5066	-6	30396	-6	182376
18	870	3/4	653	-8	5224	-8	41792
19	335	1	335	-9	3015	-9	27135
20	0	1/4	0	-10	0	-10	0
A¢		f(A)=	33683		-67087		
B				f(M)=	-16312	f(I)=	593290

~~A_{WP}~~ OR $\nabla = \frac{4}{3} S f(A) = 1,931,046$ $\Delta = 55173$

~~LCF~~ OR $LCB = S\left[\frac{f(M)}{f(A)}\right] = -20.7$

~~I_x~~ OR ~~35~~ $I_x = \frac{4}{3} S^3 f(I) = 1,771,802,560$ @54,400 T.

$J_Y = J_X - A_{WP}(LCF)^2 =$

$I_Y = I_X - \nabla(LCB)^2 = 1,750,042,560$

COMPUTATION OF J_y

$$J_y = 420 \times MTI \times L$$

$$J_y = 420 \times 6628 \times 860$$

$$J_y = 2,394,033,600$$

NOTE: WHERE MTI WAS NOT AVAILABLE, J_y WAS COMPUTED ON FORM ON PAGE 56.

APPENDIX D

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